

# A Monte Carlo study on the production scale and internal structure of jets in high energy collisions

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The production scale and internal structure of jets produced in high energy collisions are studied using Jetset 7.4 and Herwig 5.9 Monte Carlo generators. Two scales are found. One is the *jet-development scale*, which determines the size of the jet developed from a mother-parton. The other one is the *jet-production scale*, the jets produced with this scale are the most consistent with QCD jet-production dynamics and will provide the most reliable dynamical information about their mother-partons.

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## I. INTRODUCTION

The basic theory of strong interaction — Quantum Chromo-Dynamics (QCD) possesses the special property of both asymptotic freedom and color confinement. For this reason the partons (quarks and/or gluons) produced in high energy collisions have to be turned into hadrons before they can be observed in experiments. When the virtuality  $Q$  of a parton is high enough, the produced hadrons will preserve the momentum of the mother-parton, representing themselves as a cone around the moving direction of mother-parton and is referred to as *jet of hadrons*, or simply *jet*. Jets, being experimentally observable, are widely used as a tool for the experimental investigation of the physical properties of partons and their interaction dynamics.

In 1975 a two-jet structure was observed in  $e^+e^-$  annihilation experiments at  $\sqrt{s} \leq 6$  GeV [1]. This has been taken as the experimental confirmation of the production of a pair of quark-antiquark, moving back to back in  $e^+e^-$  collisions, as predicted by the parton model [2].

As energy increases the quark or anti-quark can emit a hard, *i.e.* high transverse momentum, gluon producing a third jet. This astonishing prediction of QCD was confirmed by experiments, when a third jet was observed in  $e^+e^-$  collisions at  $\sqrt{s} = 17 - 30$  GeV [3]. This observation has been recognized as the first experimental evidence of gluon.

The situation in hadron-hadron or nucleus-nucleus collisions are somewhat more complicated due to the existence of a large background. However, since the basic interaction — QCD is the same, the partons produced in these collisions, if having high enough transverse momenta, will also produce jets. The production of jets in hadron-hadron collisions was widely studied in the 80 – 90<sup>th</sup> of the last century [4, 5] and has been used as an effective way for extracting the strong coupling constant  $\alpha_s$  [6].

In last century the nucleus-nucleus (heavy ion)

collisions were performed in SPS at CERN. The corresponding experiments were fixed target ones with  $\sqrt{s_{NN}}$  being lower than 20 GeV. No jet production is expected to be observable at these energies. Only starting from this century, when the first heavy ion collider RHIC at BNL successfully run Au-Au collision at  $\sqrt{s_{NN}} = 200$  GeV, jet production becomes available. Jet physics is an important part of RHIC program. The observation of energy lose [7] of hard jets passing through the medium produced in the collision is one of the main achievements of RHIC, which is referred to as jet quenching, and is taken as one of the signals for the formation of a hot dense matter in relativistic heavy ion collisions [8]. Furthermore, jet is recognized as a powerful tool for studying the properties of the produced new form of matter [9].

In view of the highly importance of jet physics, it is necessary to study the definition and structure of jet in more detail. These are the aim of the present paper.

## II. THE SCALES IN JET DEFINITION

The definition of jet depends on scale.

Theoretically [10] jet is defined as a certain fraction  $\varepsilon$  of energy deposited in a cone with opening angle  $\delta$  around some axis. Here  $\varepsilon$  and  $\delta$  determine the scale of jet.

Experimentally, jets can be identified through some jet-finding process, *e.g.* the Jade [11] or Durham [12] algorithm. In these processes there is a parameter  $y_{\text{cut}}$ , which in case of the Durham algorithm is related to the cut in relative transverse momentum  $k_t$  [13] [14]

$$k_{t\text{cut}} = \sqrt{y_{\text{cut}}} \cdot \sqrt{s}, \quad (1)$$

where  $s$  is the c.m. energy squared. The relative transverse momentum  $k_t$  between two particles  $i$  and  $j$  is defined as [13]

$$k_{tij} = 2 \min(E_i, E_j) \sin\left(\frac{\theta_{ij}}{2}\right). \quad (2)$$

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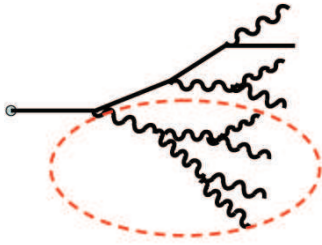


FIG. 1: A schematic sketch of the development of parton shower from a mother quark or anti-quark.

When a particle has a  $k_t$  relative to an existing jet smaller than  $k_{t\text{cut}}$  then it is grouped into this jet. The value of  $k_{t\text{cut}}$  is the scale of jet.

Alternatively, jets can also be identified by the cone algorithm [15]. Fixing some direction as jet axis, the *pseudo-rapidity*  $\eta$  and *azimuthal angle*  $\varphi$  plane are constructed, where  $\eta$  is defined as  $\eta = -\frac{1}{2} \ln \tan \theta$ ,  $\theta$  is the angle with jet axis. A parameter  $R = \sqrt{\eta^2 + \varphi^2}$  is defined for each particle. Particles with  $R \leq R_0$  is identified as a jet, where  $R_0$  is the scale of jet.

In relativistic heavy ion experiments, people usually use a high transverse momentum particle as trigger to define a jet [16]. The threshold of the trigger momentum is the scale of jet.

In all the above cases it is generally believed that how to choose the scale for jet production is a matter of definition or a matter of taste. The *jet production scale* is considered to be arbitrary in a large extent. Different scales give rise to different jets and you can choose one within a wide range that fits your requirement.

In the present paper this problem will be revisited. We will try to answer the question: in which sense can we consider the scale of jet as arbitrary, and whether or not there is a *most reasonable* scale of jet that is consistent with the physics of jet production and development in QCD.

### III. THE PRODUCTION AND DEVELOPMENT OF JET

For simplicity, let us take  $e^+e^-$  collision as example.

According to QCD the quark (anti-quark) produced in high energy  $e^+e^-$  collision will emit gluon, and the emitted gluon will in turn emit further gluons or, with lower probability, convert into quark-antiquark pairs. In this way, a parton shower is produced, turning finally into hadronic jet. The development of a parton shower from a mother-quark, or anti-quark, is sketched schematically in Fig. 1. (The other parton shower produced by the accompanied anti-quark, or quark, in the collision is not shown in the figure.) The development of parton shower is above a scale  $Q \geq Q_0 \sim 1$  GeV, which is the scale discriminating perturbative and non-

perturbative QCD. When the virtuality  $Q$  is smaller than  $Q_0$ , perturbative calculation becomes unapplicable and the partons hadronize into final state hadrons non-perturbatively.

It can be seen from the figure that the first emitted gluon will develop into a sub-parton-shower, surrounded in the figure by a dashed ellipse. When the transverse momentum of the gluon is low this sub-parton-shower will be mixed with the other partons and represent itself as a part of a unique jet. However, when the transverse momentum of the gluon is high enough, the produced sub-parton-shower might be separated from the main part, forming by itself a new jet. The transverse momentum cut that determines whether the gluon can be considered as having produced a separate jet or as being melted in the unique jet is the *scale for the production of jet*.

Is there any value which is the most reasonable scale for the production of jet according to the jet production and development processes in QCD?

To answer this question consider the development of a jet from a mother-parton through parton shower. According to the symmetry of QCD Lagrangian, the parton shower has only one privileged direction, *i.e.* the direction of the mother-parton momentum. Therefore, dynamically a single jet should possess an axial symmetry.

Then let us consider the dynamical symmetry of the  $q\bar{q}g$  system, *i.e.* the quark anti-quark together with the first emitted gluon, produced in  $e^+e^-$  collision. This is the basis of a 3-jet event. According to the symmetry of QCD Lagrangian, the gluon emission is isotropic, so the dynamical symmetry of 3-jet events is spherical.

Therefore, the most reasonable scale for jet-production is characterized by the dynamical symmetry of a single jet being axial, while that of the 3-jet events being spherical.

In a 2-jet event there are two jets flying back to back with a unique axis. Therefore, the dynamical symmetry of 2-jet events is the same as that of a single jet, *i.e.* axial.

However, in order to check the dynamical symmetry of a system, the final state particle distributions could not be used. The symmetry properties of the final state particle distributions are controlled not only by the dynamics of elementary processes but also by the process of hadronization and kinematics. They are strongly influenced by the latter, which will enshroud the dynamical symmetry of the system. In order to study the dynamical symmetry of a system, getting rid of the influence of hadronization and kinematics, the symmetry of *dynamical fluctuations* should be used instead of that of the final state *particle distributions*.

The jets from 2-jet events possessing axial symmetry with respect to dynamical fluctuations have been studied in Ref. [17] and have been given the name — *circular jet*. The scale of circular jet is found to be around  $k_{t\text{cut}}^{\text{circ}} = 6.32 \pm 0.03$  GeV for the 2-jet sample from Jetset 7.4 while  $4.28 \pm 0.02$  GeV

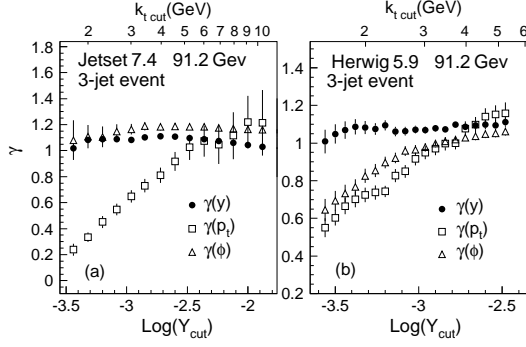


FIG. 2: The variation of the parameter  $\gamma$  with  $y_{\text{cut}}$  ( $k_{t\text{cut}}$ ) in 3-jet events from (a) Jetset 7.4 and (b) Herwig 5.9 at  $\sqrt{s}=91.2$  GeV;

for that from Herwig 5.9.

In order to check the dynamical symmetry of 3-jet events, two event samples each with 2,000,000  $e^+e^-$  events are generated from Jetset 7.4 and Herwig 5.9 at  $\sqrt{s}=91.2$  GeV, respectively. In Fig's. 2 (a) and (b) are shown the variation with  $y_{\text{cut}}$  ( $k_{t\text{cut}}$ ) of the three parameters of dynamical fluctuations —  $\gamma_y, \gamma_{p_t}, \gamma_\varphi$  (cf. Appendix) in the 3-jet events from the above-mentioned two samples. It can be seen that at  $k_{t\text{cut}}^{3\text{-jet}} = 6.03 \pm 0.06$  GeV for Jetset 7.4 and  $3.98 \pm 0.03$  GeV for Herwig 5.9 the three  $\gamma$ 's go together and the system possesses spherical symmetry.

Thus we see that the jet produced at the scale  $4 \sim 6$  GeV possesses the dynamical symmetry expected by QCD. This is the appropriate or *most reasonable* scale of jet production that is consistent with the dynamics of jet production and development in QCD.

#### IV. THE LONGITUDINAL AND TRANSVERSE DISTRIBUTIONS OF PARTICLES INSIDE JETS

Let us now turn to discuss the particle distributions inside jet obtained from the Durham jet-algorithm with various  $k_{t\text{cut}}$ 's. For this purpose 2- and 3-jet events are selected from the two event samples from Jetset 7.4 and Herwig 5.9 mentioned above for six values of  $k_{t\text{cut}}$ :  $k_{t\text{cut}} = 2, 4, 6, 8, 10, 12$  GeV. Then one jet is taken out from each event and the internal structure of this jet is studied.

Firstly, the momenta of all the particles in the jet are summed up to  $\mathbf{p}_{\text{jet}}$ , defined as the *jet momentum*. The direction of  $\mathbf{p}_{\text{jet}}$  is defined as the *longitudinal* direction and the directions perpendicular to it are the *transverse* directions. The rapidity  $y$  and transverse momentum  $p_t$  are defined along these directions as usual.

In Fig. 3 are shown the rapidity distributions inside a single jet in 2-jet events corresponding to six  $k_{t\text{cut}}$  values. It can be seen that the rapidity distribution increases sharply from  $y = 0$  to 1, then turns to a slower rise until a maximum pick around  $y = 2-4$  is reached. The pick moves leftward as the

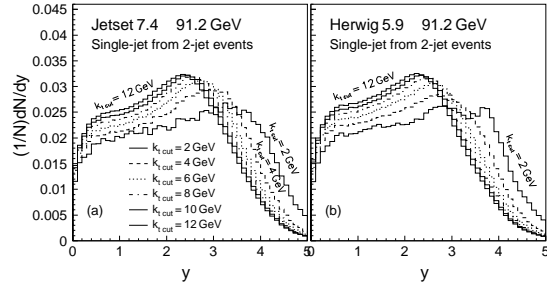


FIG. 3: The rapidity distribution inside a single jet in 2-jet events corresponding to six  $k_{t\text{cut}}$  values, from (a) Jetset 7.4 and (b) Herwig 5.9 at  $\sqrt{s}=91.2$  GeV.

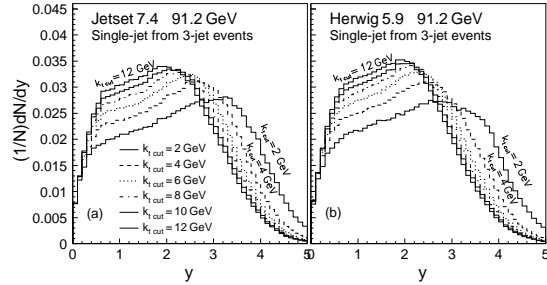


FIG. 4: The rapidity distribution inside a single jet in 3-jet events corresponding to six  $k_{t\text{cut}}$  values, from (a) Jetset 7.4 and (b) Herwig 5.9 at  $\sqrt{s}=91.2$  GeV.

increasing of  $k_{t\text{cut}}$ . The rapidity distributions inside a single jet in 3-jet events, shown in Fig. 4, have a similar behavior.

The particles with rapidity at or to the right of the pick are the leading particles of the corresponding jets. Their momenta along the jet axis, taking as  $z$  axis, are about  $p_z \geq 10$  GeV/c. This can be used as the scale for triggering jets.

Contrary to the dependence of rapidity distribution on  $k_{t\text{cut}}$ , the relative transverse momentum  $k_t$  distribution inside a single jet turns out to be insensitive to  $k_{t\text{cut}}$ , cf. Fig.'s 5 and 6. Here we use the Jade definition for  $k_t$  [11]

$$k_{t\text{ij}}^{\text{jade}} = 2\sqrt{E_i E_j} \sin\left(\frac{\theta_{ij}}{2}\right), \quad (3)$$

which is basically the same as Durham  $k_t$  but is symmetric with respect to  $i$  and  $j$ . This definition is more natural when we study the internal structure of jet, instead of absorbing a particle into the main part of jet in jet-algorithm.

The insensitivity of  $k_t$  distribution on  $k_{t\text{cut}}$  means that there is a scaling property in the transverse direction inside jet. Note that the value of  $k_{t\text{cut}}$  is the parameter, which determines how to group particles to jets. A particle that has a  $k_t$  with respect to an existing jet less than  $k_{t\text{cut}}$  belong to that jet, while those with  $k_t$  larger than  $k_{t\text{cut}}$  belong to another jet. Thus  $k_{t\text{cut}}$  is the lower limit of the distance between two jets and at the same time the upper limit of the size of jet. Therefore, it is natural to expect that the size of jet will increase with the increasing of

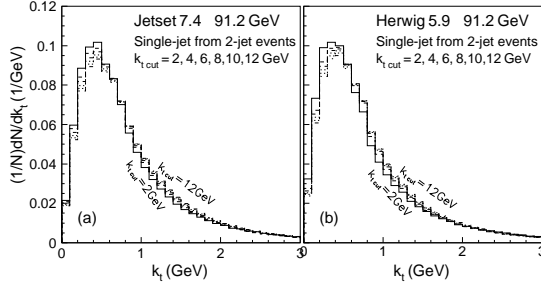


FIG. 5: The relative transverse momentum  $k_t$  distribution inside a single jet in 2-jet events, corresponding to six  $k_{t\text{cut}}$  values, from (a) Jetset 7.4 and (b) Herwig 5.9 at  $\sqrt{s}=91.2$  GeV.

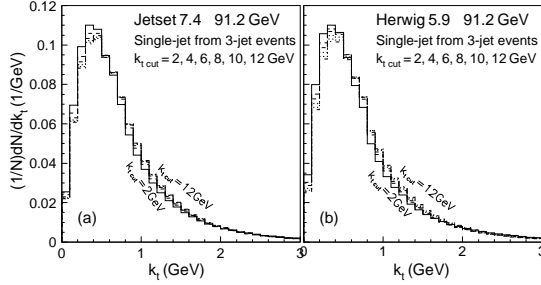


FIG. 6: The relative transverse momentum  $k_t$  distribution inside a single jet in 3-jet events, corresponding to six  $k_{t\text{cut}}$  values, from (a) Jetset 7.4 and (b) Herwig 5.9 at  $\sqrt{s}=91.2$  GeV.

$k_{t\text{cut}}$ . The transverse scaling shown in Fig's. 5, 6 is contrary to this expectation.

In order to be more intuitive, we show in Fig. 7 the scattering plot of the particles in two neighboring jets of 3-jet events, corresponding to three  $k_{t\text{cut}}$  values. Two jets  $a$  and  $b$  are *neighboring* means that the distance  $\sqrt{(p_{t_{ax}} - p_{t_{bx}})^2 + (p_{t_{ay}} - p_{t_{by}})^2}$  of their axes being the smallest among all the jet-distances in the event. A reference frame is constructed, using the line  $p_{t_a} p_{t_b}$  as  $x$  axis and putting the origin at the middle point of the line segment  $\overline{p_{t_a} p_{t_b}}$ . In order to increase statistics 10 events with two neighboring jets separated for the same distance

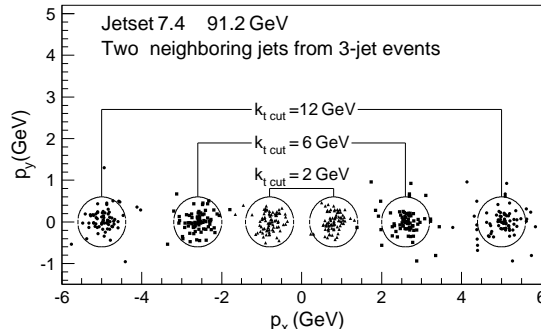


FIG. 7: The scattering plots of the particles in two neighboring jets of 3-jet events, corresponding to three  $k_{t\text{cut}}$  values.

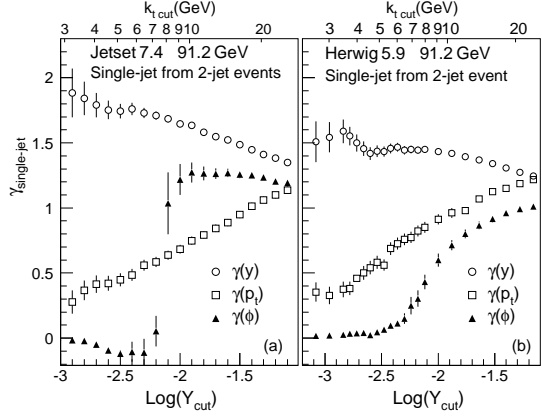


FIG. 8: The variation of the parameters  $\gamma(y)$ ,  $\gamma(p_t)$ ,  $\gamma(\varphi)$  with  $y_{\text{cut}}$  ( $k_{t\text{cut}}$ ) in a single jet of 2-jet events from (a) Jetset 7.4 and (b) Herwig 5.9 at  $\sqrt{s}=91.2$  GeV, calculated in jet-axis frame.

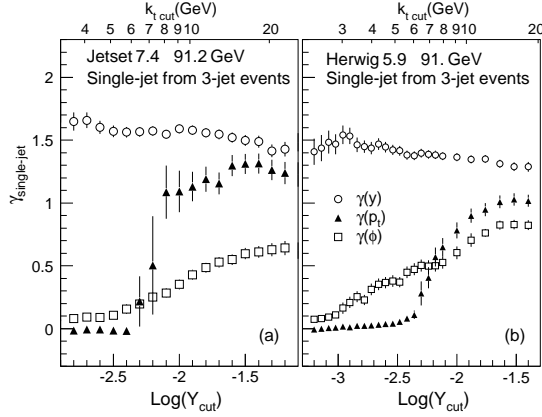


FIG. 9: The variation of the parameters  $\gamma(y)$ ,  $\gamma(p_t)$ ,  $\gamma(\varphi)$  with  $y_{\text{cut}}$  ( $k_{t\text{cut}}$ ) in a single jet of 3-jet events from (a) Jetset 7.4 and (b) Herwig 5.9 at  $\sqrt{s}=91.2$  GeV, calculated in jet-axis frame.

are selected and the results from these events are superposed in Fig. 7. It can clearly be seen from the figure that the size of jet is universal, independent of the distance between jets.

## V. THE DYNAMICAL FLUCTUATIONS INSIDE JETS

The transverse structure of jet has scaling property means that the scale for jet production can be chosen within a wide range and jets with a unique relative transverse momentum  $k_t$  distribution will still be obtained. This phenomenon supports the approach of choosing jet production scale arbitrarily within a wide range.

Then how to understand the speciality of the *most reasonable* scale of jet production at 4–6 GeV? This scale is related to the QCD dynamics of jet production and development. Its special role will be presented only in the dynamical fluctuations but not in the particle distributions, *cf.* the discussion

in Sec. III above.

In Fig. 2 of Ref. [17] and Fig. 2 of the present paper the  $\gamma$  parameters of event-dynamical-fluctuations have been shown, where the factorial moments are calculated for the 2- or 3-jet events in the *event-thrust frame*. Now we want to study the dynamical fluctuations inside jets, so we take the *particles in a single jet* and calculate the factorial moments, *cf.* Appendix, of these particles in the *frame with jet axis as the  $z$  axis*. The results are shown in Fig's. 8 and 9, where Fig's. 8 is the variation of the parameters  $\gamma(y)$ ,  $\gamma(p_t)$ ,  $\gamma(\varphi)$  with  $y_{\text{cut}}$  ( $k_{t\text{cut}}$ ) in a single jet of 2-jet events and Fig's. 9 is those in a single jet of 3-jet events from Jetset 7.4 and Herwig 5.9 at  $\sqrt{s}=91.2$  GeV, respectively. All the figures are calculated in the jet-axis frame.

A transition point can clearly be seen from the figures, which is located at  $k_{t\text{cut}}^{\text{dynam}} = 6.84 \pm 0.20$  GeV (Jetset),  $5.62 \pm 0.19$  GeV (Durham) for single jet from 2-jet events; and  $k_{t\text{cut}}^{\text{dynam}} = 6.10 \pm 0.19$  GeV (Jetset),  $5.82 \pm 0.20$  GeV (Durham) for that from 3-jet events. At this point the  $\gamma(\varphi)$  has a sudden rise. This sudden rise from  $\gamma(\varphi) \sim 0$  to a saturation value means that the gluons emitted with  $k_t < k_{t\text{cut}}^{\text{dynam}}$  will be simply included in the parton shower, having no dynamical fluctuation in azimuth. On the contrary, the gluons emitted with  $k_t > k_{t\text{cut}}^{\text{dynam}}$  will be *hard* enough to develop as a separate jet and the dynamical fluctuations in azimuth appear.

Table 1 The scales for jet production

	Jetset 7.4	Herwig 5.9
$k_{t\text{cut}}^{\text{circ}}$ (GeV)	$6.32 \pm 0.03$	$4.28 \pm .02$
$k_{t\text{cut}}^{\text{3-jet}}$ (GeV)	$6.03 \pm 0.06$	$3.98 \pm 0.03$
$k_{t\text{cut}}^{\text{dynam}}$ (GeV) from 2-jet events	$6.84 \pm 0.20$	$5.62 \pm 0.19$
$k_{t\text{cut}}^{\text{dynam}}$ (GeV) from 3-jet events	$6.10 \pm 0.19$	$5.82 \pm 0.20$

In Table I are listed the scales found in Ref. [17], in Sec. III and this section of the present paper. The coincidence of the scales obtained from different origins is remarkable. All of them are related to jet production, so we give them a unique name — *jet production scale* and denote them as  $k_{t\text{cut}}^{\text{jet-prod}}$ .

## VI. CONCLUSION

As discussed in Sec. III, a highly-virtual parton will emit gluons and develop to a parton shower, hadronizing eventually to jet. The jets are defined with a scale  $k_{t\text{cut}}$ , which can take values in a wide range, *cf.* Fig. 7. When  $k_{t\text{cut}}$  takes a large value, *e.g.*  $k_{t\text{cut}} = 12$  GeV, the emitted gluon might in principle have a  $k_t$  value almost as large as this value, *i.e.* only a little bit smaller than 12 GeV, and still be a “daughter” of the mother parton. In other words, when  $k_{t\text{cut}} = 12$  GeV the development of a parton to jet might be extended to a scale about equal to 12 GeV. However, it does not act as that. The development of a parton to jet stops at a scale much

smaller than 12 GeV, *cf.* Fig's. 5, 6. We will refer to this scale as the *jet development scale* and denote it as  $k_{t\text{cut}}^{\text{jet-dev}}$ .

This scale measures the largest boundary of a parton shower developed from a mother parton. Since the  $k_t$  distribution tends to zero exponentially, *cf.* Fig's. 5, 6, we take the  $k_t$  value, where the probability density reduces for an order of magnitude from its maximum, as a measure of  $k_{t\text{cut}}^{\text{jet-dev}}$ , *i.e.* we define  $k_{t\text{cut}}^{\text{jet-dev}}$  through

$$\frac{1}{N} \frac{dN}{dk_t} (k_{t\text{cut}}^{\text{jet-dev}}) = 0.1 \cdot \left( \frac{1}{N} \frac{dN}{dk_t} \right)_{\text{max}}. \quad (4)$$

Taking the 24  $k_t$  distributions obtained for single jets in 2- and 3-jet events from two MC generators — Jetset 7.4 and Herwig 5.9, for 6  $k_{t\text{cut}}$  values —  $k_{t\text{cut}} = 2, 4, 6, 8, 10, 12$  GeV, shown in Fig's. 5, 6, as a unique sample and calculate  $k_{t\text{cut}}^{\text{jet-dev}}$  from this sample according to Eq. (4), we get

$$k_{t\text{cut}}^{\text{jet-dev}} = 1.81 \pm 0.10 \text{ GeV}. \quad (5)$$

The highly convergent result from events with different numbers of jet, from different generators and from various values of  $k_{t\text{cut}}$  is remarkable. It shows that the dynamics for the development of a parton to jet is universal. It has a scale  $k_{t\text{cut}}^{\text{jet-dev}}$ . All the jets produced with various  $k_{t\text{cut}}$  is of the same scale.

However, the jets produced with different  $k_{t\text{cut}}$  are not totally equivalent. There is a jet-production scale  $k_{t\text{cut}}^{\text{jet-prod}}$ , the jets produced with  $k_{t\text{cut}} = k_{t\text{cut}}^{\text{jet-prod}}$  are the most consistent with QCD jet-production dynamics in the following sense:

- The jets in 2-jet events produced with  $k_{t\text{cut}} = k_{t\text{cut}}^{\text{jet-prod}}$  is circular with respect to dynamical fluctuations, *i.e.* the dynamical fluctuations in these jets is circular in the transverse plane. These jets have axial dynamical symmetry as expected by QCD.
- When a third jet is produced with  $k_{t\text{cut}} = k_{t\text{cut}}^{\text{jet-prod}}$ , the resulting 3-jet system is isotropic in consistent with QCD expectation.
- Inside a single jet when a gluon is emitted with  $k_t = k_{t\text{cut}}^{\text{jet-prod}}$ , dynamical fluctuations in azimuthal angle appears suddenly, which means that this gluon is no more a part of the original jet but is the “mother” of a new jet.

We conclude that there exist two scales in jet physics. One is the *jet-development scale*  $k_{t\text{cut}}^{\text{jet-dev}}$  ( $\sim 1.8$  GeV), which determines the size of the jet developed from a mother-parton. The other one is the *jet-production scale*  $k_{t\text{cut}}^{\text{jet-prod}}$  ( $\sim 4-6$  GeV), the jets produced with this scale are the most consistent with QCD jet-production dynamics.

We can identify jets with various scales  $k_{t\text{cut}}$  in a wide range and universal jets with the same transverse distribution will be obtained. Among them

only those identified with  $k_{\text{tcut}} = k_{\text{tcut}}^{\text{jet-prod}}$  are the most consistent with QCD jet-production dynamics. It is the *best representative* of the mother parton. When we use jets to study the properties of partons, those identified with  $k_{\text{tcut}} = k_{\text{tcut}}^{\text{jet-prod}}$  will provide the most reliable dynamical information about their mother-partons.

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## APPENDIX: DYNAMICAL FLUCTUATIONS

The dynamical fluctuations can be characterized by the anomalous scaling of normalized factorial moments (NFM) [18]:

$$F_q(M) = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m-1) \cdots (n_m-q+1) \rangle}{\langle n_m \rangle^q} \propto (M)^{\phi_q} \quad (M \rightarrow \infty), \quad (\text{A.1})$$

where a region  $\Delta$  in 1-, 2- or 3-dimensional phase space is divided into  $M$  cells,  $n_m$  is the multiplicity in the  $m$ th cell, and  $\langle \cdots \rangle$  denotes vertically averaging over the event sample.

When the fluctuations exist in higher-dimensional (2-D or 3-D) space, the projection effect [19] will cause the second-order 1-D NFM to go to saturation according to the rule [20]:

$$F_2^{(a)}(M_a) = A_a - B_a M_a^{-\gamma_a}, \quad (\text{A.2})$$

where  $a = 1, 2, 3$  denotes the different 1-D variables. The parameter  $\gamma_a$  describes the rate of approach to saturation of the NFM in direction  $a$  and is the most important characteristic for the higher-dimensional dynamical fluctuations. If  $\gamma_a = \gamma_b$ , the fluctuations are isotropic in the  $a, b$  plane. If  $\gamma_a \neq \gamma_b$ , the fluctuations are anisotropic in this plane.

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